Question Paper

Physics Single Correct (Maximum Marks: 15)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A gas expands with temperature according to the relation $V = KT^{2/3}$. What is the work done when the temperature changes by $30^{\circ}C$ for 1 mole of gas?

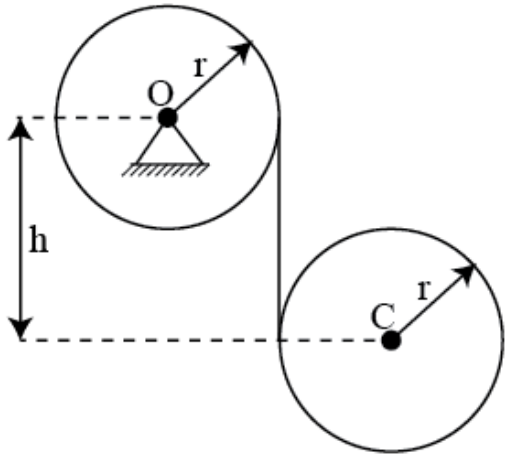
A. $10R$
B. $20R$
C. $30R$
D. $40R$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Consider the combination of two discs, then what will be the velocity of falling disc centre as a function of h. Both discs are identical and string does not slip relative to disc.



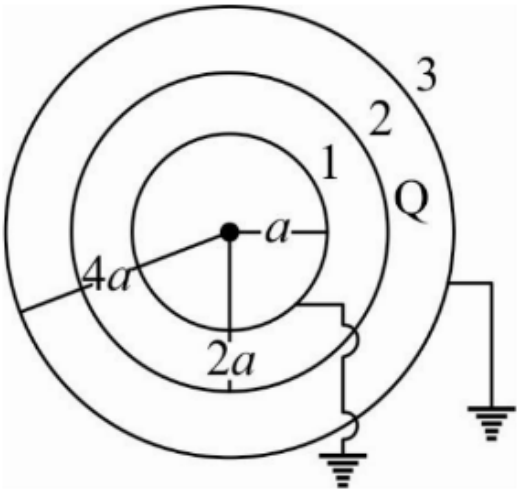
A. $\sqrt{\frac{8gh}{5}}$
B. $\sqrt{\frac{6gh}{5}}$
C. $\sqrt{\frac{gh}{5}}$
D. $\sqrt{gh}$

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

There are three conducting spherical shells. The shells 1 and 3 are earthed. The potential (in volt) of the middle conducting shell carrying a charge $Q(= 24\pi\varepsilon_0 a)$ is :



A. 1
B. 2
C. 3
D. None of these

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A particle of mass ' m ' is rotating in a circle of radius ' r ' with power $P = mk^2r^2t$. If the particle starts its motion from rest, then its centripetal acceleration is :

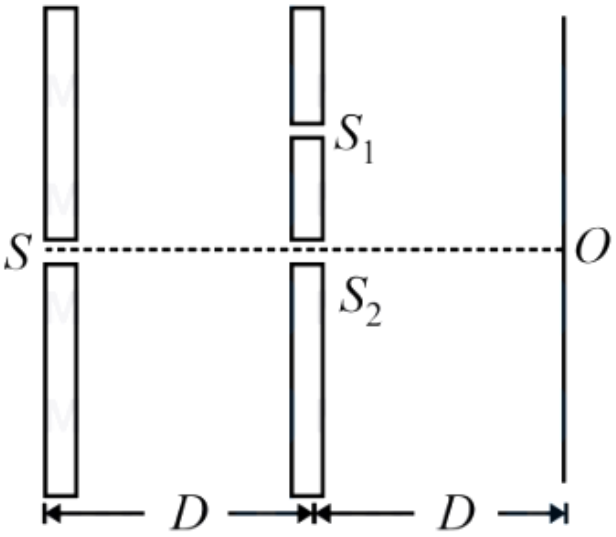
A. kr^2t^2
B. k^2r^2t
C. k^2rt^2
D. kr^2t

Question No. 5

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Two ideal slits S_1 and S_2 are at a distance d apart and illuminated by light of wavelength λ, passing through an ideal source slit S, placed on the line through S_2, as shown. The distance between the planes of slits and the source slit is D. A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is



A. $\sqrt{\frac{3\lambda d}{2}}$
B. $\sqrt{\lambda D}$
C. $\sqrt{\frac{\lambda D}{2}}$
D. $\sqrt{3\lambda D}$

Physics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An object and a screen are kept at a distance of 120 cm. A lens of focal length 22.5 cm is kept between them so that a real image is formed on the screen. Find the possible location(s) of the lens.

A. 90 cm from object
B. 30 cm from object
C. 40 cm from object
D. 80 cm from object

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A process for monoatomic gas is defined by $V = aT^2$, where a is a constant. In this process temperature of 1 mole of a gas is increased by 100 K; then (V = Volume, T = Temperature)

A. Molar heat capacity of this process is negative.
B. The work done by the gas in the process is 250R.

C. The work done by the gas in the process is 200R.

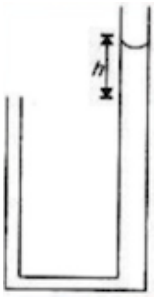
D. The work done is given by $\int_{V_1}^{V_2} PdV$ in which P is a function of volume.

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two capillaries of small cross section are connected as shown in the figure. The right tube has cross sectional radius R and left one has a radius of $r(< R)$. The tube of radius R is very long where as the tube of radius r is of short length. Water is slowly poured in the right tube. Contact angle for the tube wall and water is $\theta = 0^\circ$. Let h be the height difference between water surface in the right and left tube. Surface tension of water is T and its density is ρ. Mark the CORRECT option(s)



A. The value of h if the water surface in the left tube is found to be flat is $h = \frac{2T}{R\rho g}$

B. The value of h if the water surface in the left tube is found to be flat is $h = \frac{T}{R\rho g}$

The maximum value of h for which water will not flow out of the left tube is

C. $\frac{2T}{\rho g}\left(\frac{r+R}{rR}\right)$

The maximum value of h for which water will not flow out of the left tube is

D. $\frac{T}{\rho g}\left(\frac{r+R}{rR}\right)$

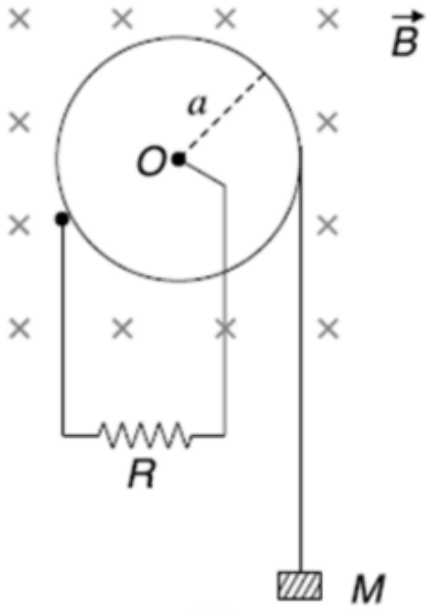
Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A metallic pulley is in the shape of a disc of radius a. It can rotate freely about a horizontal axis passing through its centre. The moment of inertia of the pulley about this axis is I. A light string is tightly wrapped around the pulley with its one end connected to a block of mass M. The centre of the pulley and its circumference are connected to a resistance R as shown. The contact of resistance at the circumference does not cause any friction. A uniform magnetic field B is switched on which is parallel to the axis of rotation of the pulley (see Figure). The mass M is allowed to fall. Assume that resistivity of the material of the pulley is negligible.

The acceleration of the block of mass M at the instant its velocity becomes v_0 is



$\frac{Mga^2-\frac{B^2a^4v_0}{4R}}{I+Ma^2}$.

A. Assuming that the block can fall through a large distance, the terminal speed (v_T) that it will acquire is $\frac{4MgR}{B^2a^2}$.

B. The rate of change of kinetic energy of the pulley at the instant when the speed of the falling block is $\frac{v_T}{2}$ is equal to $\frac{IM^2g^2R}{B^2a^2(I+Ma^2)}$.

C. The rate of change of kinetic energy of the pulley at the instant when the speed of the falling

D. block is $\frac{v_T}{2}$ is equal to $\frac{IM^2g^2a^2}{B^2R(I+Ma^2)}$.

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A pulse is started at a time $t = 0$ along the +x direction on a long, taut string. The shape of the pulse at $t = 0$ is given by function $f(x)$ with

$$f(x) = \begin{cases} \frac{x}{4} + 1 & \text{for} \quad -4 < x \leq 0 \\ -x + 1 & \text{for} \quad 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

here f and x are in centimeters. The linear mass density of the string is $50\ \text{g/m}$ and it is under a tension of 5 N.

A. The shape of the string is drawn at $t = 0$ then the area of the pulse enclosed by the string and the x-axis is $2.5\ \text{cm}^2$.

B. The shape of the string is drawn at $t = 0$ then the area of the pulse enclosed by the string and the x-axis is $5\ \text{cm}^2$.

C. The transverse velocity of the particle at $x = 13\ \text{cm}$ and $t = 0.015\ \text{s}$ will be $-250\ \text{cm/s}$

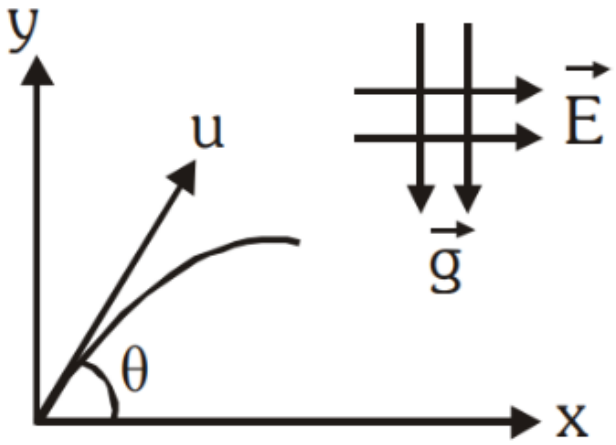
D. The transverse velocity of the particle at $x = 13\ \text{cm}$ and $t = 0.015\ \text{s}$ will be $250\ \text{cm/s}$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A particle of mass m and charge $-q$ has been projected from ground as shown in the figure below.



Mark out the correct statements (s) :-

A. The path of motion of the particle can be parabolic

B. The path of motion of the particle can be straight line

C. Time of flight of particle is $\frac{2u \sin\theta}{g}$

D. Range of motion of the particle will be less than $\frac{u^2 \sin 2\theta}{g}$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of $40\ \text{cm}$. The block just loses contact with the plank, when the latter is at momentary rest. Then :

A. The period of oscillation is $(2\pi/5)$
B. The block weights on the plank double its weight, when the plank is at one of the positions of momentary rest
C. The block weighs on the plank 1.5 times its weight halfway in the oscillation.
D. The block weighs 1.5 times its weight on the plank halfway down.

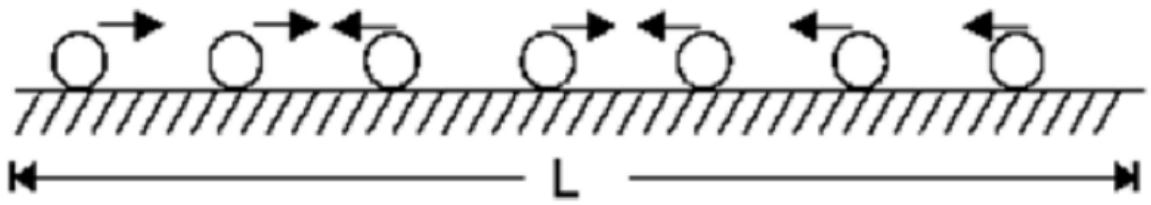
Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

There are 40 identical balls travelling along a straight line on a smooth horizontal table. All balls have equal speed $v(10\ \text{m/s})$ and each one is travelling to right or left. All collisions between the balls is head-on and perfectly-elastic. At some point in time all balls will have fallen off the table. The time at which this happens will definitely depend on initial positions of the balls. Over all possible initial positions of the balls; what is the longest amount of time that you would need to wait to ensure that the table has no more balls? Assume that length of the table is $L(120\ \text{m})$.



Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

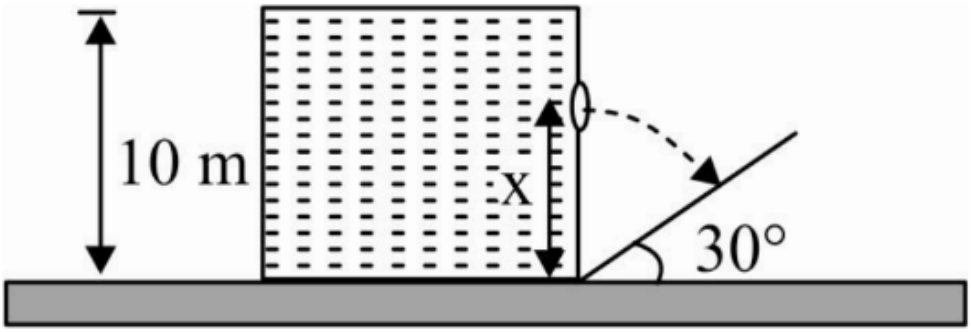
A resistance (R), inductance (L) and capacitance (C) are connected in series to an ac source of voltage v having variable frequency. If the energy delivered by the source to the circuit during one period if the operating frequency is twice the resonance frequency is $\frac{\pi R\sqrt{LC}V^2}{R^2+P(\frac{L}{C})}$ Find P.

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

A rectangular tank of height 10 m filled with water, is placed near the bottom of an incline of angle 30°. At height x from the bottom, a small hole is made as shown in figure such that the stream coming out from the hole strikes the inclined plane normally. Find the value of $0.6 \times$?

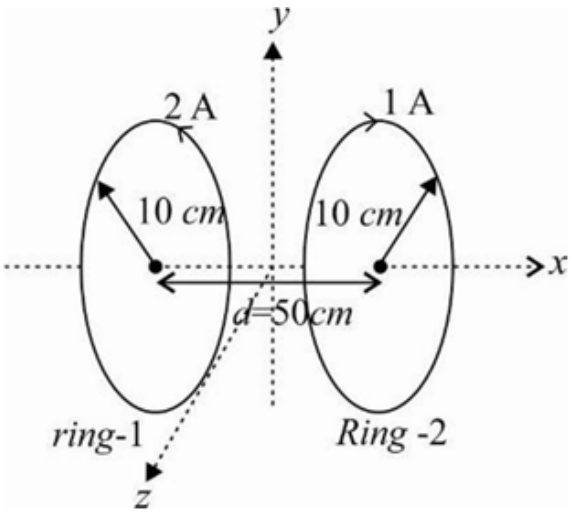


Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Two co-axial rings of same radius $R = 10$ cm are placed parallel to the $y - z$ plane, such that x-axis is the axis of the rings. Ring-1 carries a current of 2 Amperes and Ring-2 carries a current of 1 Ampere in opposite direction as shown in the figure. The separation between the rings is $d = 50$ cm . Find the magnitude of $\frac{2\left|\int_{+\infty}^{-\infty} \vec{B} \cdot d\vec{x}\right|}{\mu_0}$, where $\vec{B}$ is the net magnetic field due to both the rings at any point on the x-axis.



Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

A sphere of brass released in a long liquid column attains a terminal speed v_0. If the terminal speed attained by the sphere of marble of the same radius and released in the same liquid is nv_0, then the value of n will be $\frac{17}{11x}$ then find x ?

Given: The specific gravities of brass, marble and the liquid are $8.5, 2.5$ and 0.8 respectively.

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

A shell of radius $1\ \text{m}$ is coated with a thin layer of β^- active material. It's initial charge is zero and initial number of active atoms is $\frac{4}{3} \times 10^{12}$. If half life of decay is 1hr and all the electron are emitted with an energy of 1.44keV, find the time (in hr.) after which charge on the sphere becomes constant. Neglect the time taken by electron to return back.

Chemistry Single Correct (Maximum Marks: 15)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

In paper electrophoresis, amino acids and peptides can be separated by their differential migration in an electric field. To the center of a strip of paper, wet with buffer at pH 6, is applied a mixture of the following three peptides in a single small spot : Gly-Lys, Gly-Asp, and Gly-Ala. A positively charged electrode (anode) is attached to the left side of the paper, and a negatively charged electrode (cathode) to the right side. A voltage is applied across the ends of the paper for a time, after which the peptides have separated into three spots : one near the cathode, one near the anode and one in the center, at the location of the original spot. Which statement is incorrect?

A. Gly-Lys migrates to the cathode
B. Gly-Asp migrates to the anode
C. Gly-Ala does not migrate
D. Gly-Lys is a acidic dipeptide

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

For a certain orbital

$$\Psi(r) = \frac{1}{16\sqrt{4}a_0^{3/2}}(1-\sigma)(\sigma^2 - 8\sigma + 12)e^{-\sigma/2}\ , \sigma = \frac{2r}{na_0}\ \{a_0 = \text{ constant }\}$$

The distance of nearest radial node from nucleus is xa_0 while farthest radial node from nucleus is ya_0.

Find $(\frac{y}{x})$.

A. 4
B. 6
C. 4.5
D. 3.5

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Which of the following expressions correctly represents the equivalent conductance at infinite dilution of $Al_2(SO_4)_3$. Given that $\Lambda_{Al^{3+}}$ and $\Lambda_{SO_4^{2-}}$ are the equivalent conductances at infinite dilution of the respective ions?

A. $2\Lambda_{Al^{3+}} + 3\Lambda_{SO_4^{2-}}$

B. $\Lambda_{Al^{3+}} + \Lambda_{SO_4^{2-}}$

C. $(\Lambda_{Al^{3+}} + 3\Lambda_{SO_4^{2-}}) \times 6$

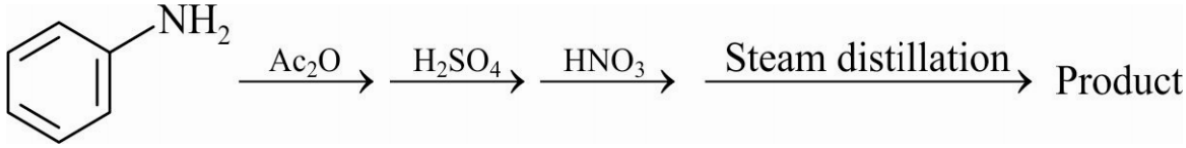
D. $\frac{1}{3}\Lambda_{Al^{3+}} + \frac{1}{2}\Lambda_{SO_4^{2-}}$

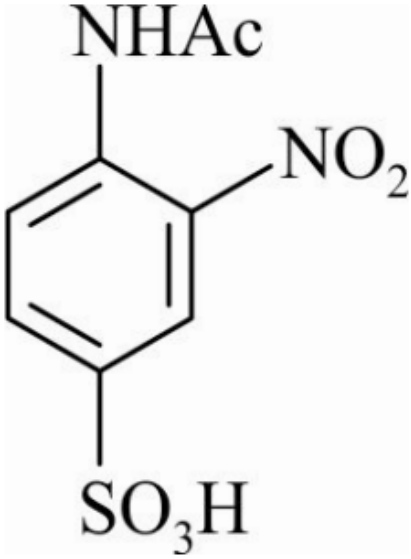
Question No. 4

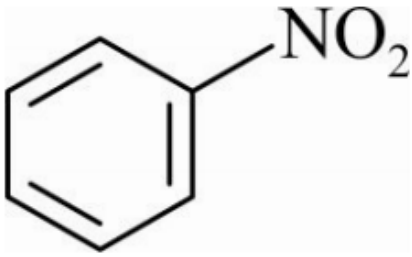
Only One Option Correct Type

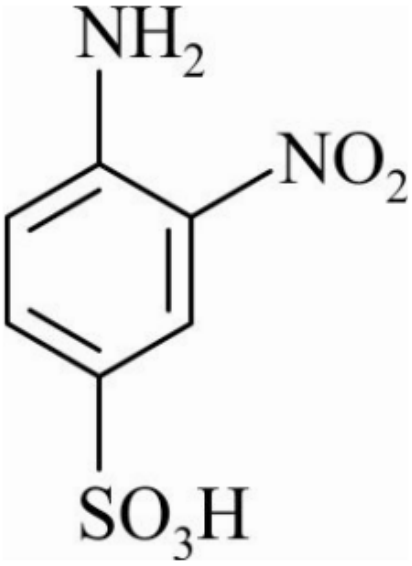
Each question has multiple options out of which ONLY ONE is correct.

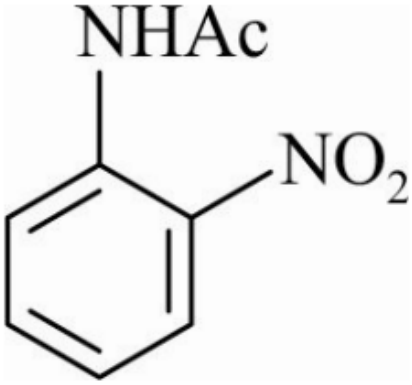
The major product obtained in the following reaction is:



A. 

B. 

C. 

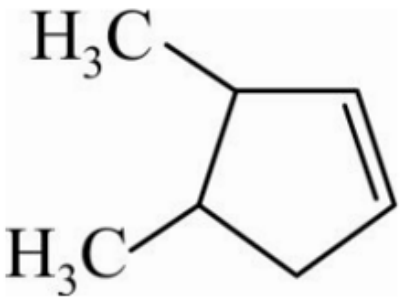
D. 

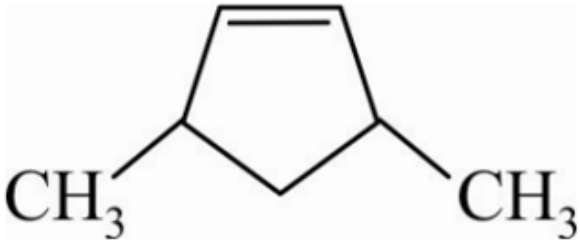
Question No. 5

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

An alkene (A) with molecular formula C_7H_{12} adds HBr to give a single alkyl halide (B)($C_7H_{13}Br$) . (A) on catalytic hydrogenation gives 1, 1-dimethyl cyclopentane. The alkene (A) is :

A. 

B. 

C. 

D. 

Chemistry Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In which of the following $\Delta S_{surrounding}$ will drive the reaction in forward direction?

A. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
B. $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$
C. $H_2O(l) \rightleftharpoons H_2O(s)$
D. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Find the correct statement(s)

A. In 1 gram of NaCl, units cells is 2.57×10^{21}
B. Total tetrahedral voids that are occupied in diamond is 50%
C. Co-ordination no. in CaF_2 is 4 : 8

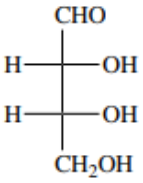
D. Co-ordination number in Na_2O is $4 : 8$

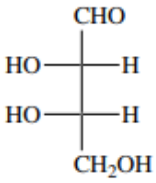
Question No. 3

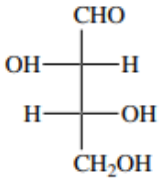
One or More Options Correct Type

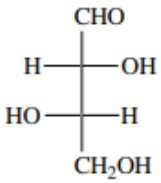
The question has multiple options out of which ONE or MORE is/are correct.

Which of the following compounds on reduction give meso-1, 2, 3, 4-butanetetrol

A.


B.


C.


D.


Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which of the following statements are not correct?

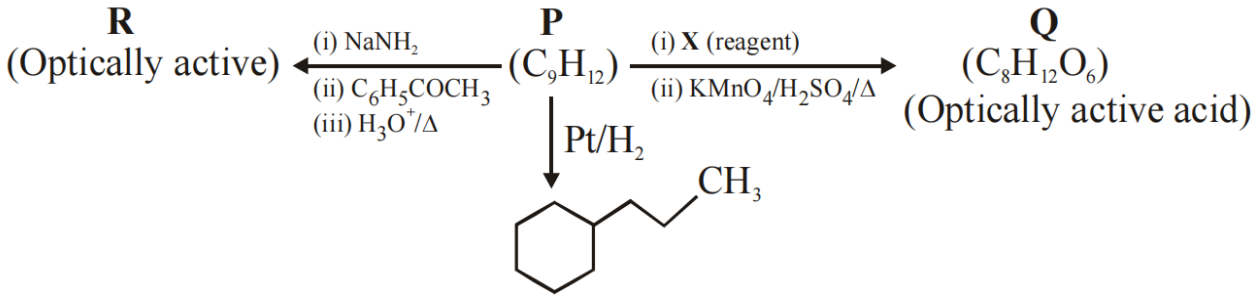
A. Beryllium, like nitrogen, forms diatomic molecule Be_2.
B. He_2 molecule does not exist but He_2^+ does exist.
C. The dipole moment of CH_3F is greater than that of CH_3Cl.
D. HBr is a stronger acid than HI because of hydrogen bonding.

Question No. 5

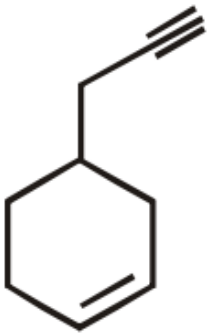
One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

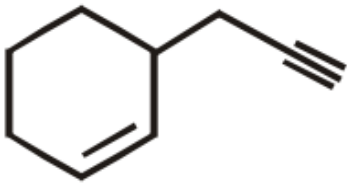
Consider the following transformations of a compound **P**.

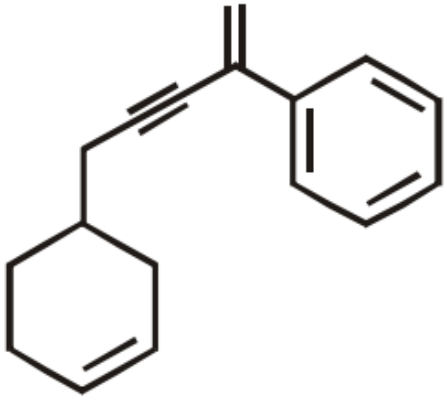


Choose the correct option(s).

A. **P** is 

B. **X** is Pd − C/ quinoline/ H_2

C. **P** is 

D. **R** is 

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Select the correct statement(s) regarding following galvanic cell taken at temperature TK.

$$Pt \mid H_2(g, 1\ bar) \left| HCl(10^{-3}\ M) \right| AgCl(s) \mid Ag(s)$$

Given that : $\frac{2.303RT}{F} = 0.06\ volt$

&$E^{o}_{Ag|AgCl|CC^-} = -0.18 volt$

A. Cell potential of this cell is 0.54 volt.
B. During discharging, H_2 (g) converts into H^+ (aq.)
C. $H_2(g)$ is obtained at cathode during working of this galvanic cell.
D. $E^\circ_{Ag^+|Ag}$ is equal to 0.78 volt if K_{sp} of AgCl is 1×10^{-10} .

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

On addition of $Na_2O_2(s)$ to aqueous $CrCl_3$, a yellow coloured chromium compound (P) is formed. (P) on reaction with H_2O_2 in presence of H_2SO_4 forms a deep violet- blue chromium compound (**Q**) which is extracted in ether. Choose the correct option(s) for (P) and (**Q**) :

A. Formation of (**Q**) from (**P**) involves a redox change.
B. Oxidation state of chromium is same in both (P) and (Q).
C. The number of oxygen atoms doubly bonded to chromium is same in both (P) and (**Q**)
D. The geometry around chromium is different in (**P**) and (**Q**) .

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases *x* times.

The value of x is

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Among the given carbonates / bicarbonates, find the value of $(\frac{x^z}{y})$

$KHCO_3, NaHCO_3, CaCO_3, Ca(HCO_3)_2$, K_2CO_3 and $Mg(HCO_3)_2$

no. of compounds almost insoluble in water = x

no. of compound which liberate CO_2 on reaction with HCl = z

no. of compounds that easily undergo thermal decomposition = y

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

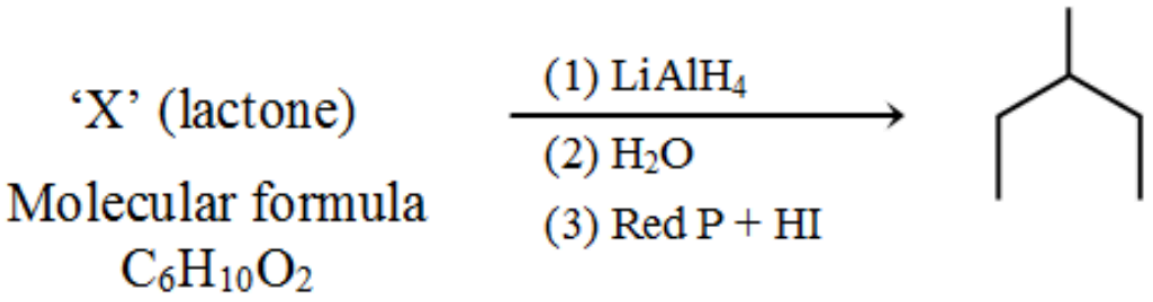
Total number of compounds with molecular formula $C_4H_8O_2$, which can reduce both Tollen's reagent and PCC is:

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

In the given reaction sequence, find all possible five and six membered lactones



Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

At $20^{\circ}C$, the vapour pressure of 0.1 molal aqueous solution of urea is $0.03\ mm$ less than that of water and the vapour pressure of 0.1 molal solution if KCl is $0.0594\ mm$ less than that of water. The apparent percentage dissociation of KCl in water at the given temperature is (Neglect the moles of solute particles in comparison to the moles of water in both solutions).

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

How many ml of $0.1M NaOH$ is added to 60ml of $0.15M H_3PO_4$ (pK_{a_1}, pK_{a_2} and pK_{a_3} for H_3PO_4 are 3,8 and 13). So, that pH of resulting buffer solution would be 8.3. ($\log 2 = 0.3$)

Mathematics Single Correct (Maximum Marks: 15)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Each of 4 boys attends a school gathering with both of his parents. The number of ways these 12 people can be divided into groups of three each group containing a boy, a male parent, a female parent such that

List-I		List-II	
(P)	No boy is with both of his parents in his group is N then N is	(1)	5
(Q)	Exactly two boys are with both their parents is N then N is	(2)	453
(R)	Two particulars boys are with both of his parents is N then N is	(3)	3
(S)	Every boy is with both his parents is N then N is	(4)	18
		(5)	1
		(6)	283

The correct option is

A. $P \rightarrow 6; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 5$
B. $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 1; S \rightarrow 5$
C. $P \rightarrow 6; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 5$
D. $P \rightarrow 2; Q \rightarrow 4; R \rightarrow 3; S \rightarrow 5$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

A. Infinitely solution
B. Three solutions
C. One solution
D. No solution

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

The solution of the differential equation $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$ is :

(Where c is arbitrary constant)

A. $\frac{1}{x+y} = x^2 + 1 + ce^x$
B. $\frac{1}{(x+y)^2} = x^2 + 1 + ce^{x^2}$
C. $\frac{1}{(x+y)^2} = x + 1 + ce^x$
D. $\frac{1}{x+y} = x + 1 + ce^{x^2}$

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Let $f''(x) < 0 \quad \forall x \in R$ and $g(x) = f(2 - x^2) + f(x^2)$ then set of possible values of x for which g(is increasing is :

A. $(-1, 0) \cup (1, \infty)$
B. $(-\infty, -1) \cup (1, \infty)$
C. $(-\infty, -1) \cup (0, 1)$
D. $(-1, 1)$

Question No. 5

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

If $x^8 - (k - 1)x^4 + 5 = 0$, then least possible integral value of ' k ' so that equation has maximum number of real roots is :

A. 4
B. 5
C. 6
D. 7

Mathematics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The correct statement(s) is/are -

The line of intersection of planes $\vec{r} \cdot \vec{n}_1 = q_1, \vec{r} \cdot \vec{n}_2 = q_2$ and $\vec{r} \cdot \vec{n}_3 = q_3, \vec{r} \cdot \vec{n}_4 = q_4$ are perpendicular if

A. $(\vec{n}_1 \cdot \vec{n}_3)(\vec{n}_2 \cdot \vec{n}_4) = (\vec{n}_1 \cdot \vec{n}_4)(\vec{n}_2 \cdot \vec{n}_3)$.

If three distinct planes $\vec{r} \cdot \vec{n}_1 = q_1, \vec{r} \cdot \vec{n}_2 = q_2, \vec{r} \cdot \vec{n}_3 = q_3$ intersect in a line which is contained by the plane $\vec{r} \cdot \vec{n}_4 = q_4$, then

B. $[\vec{n}_1 \quad \vec{n}_2 \quad \vec{n}_4]\vec{n}_3 = [\vec{n}_1 \quad \vec{n}_2 \quad \vec{n}_3]\vec{n}_4$.

If four distinct planes $\vec{r} \cdot \vec{n}_1 = q_1, \vec{r} \cdot \vec{n}_2 = q_2, \vec{r} \cdot \vec{n}_3 = q_3$ and $\vec{r} \cdot \vec{n}_4 = q_4$

C. intersect in a line, then $[\vec{n}_1 \quad \vec{n}_2 \quad \vec{n}_4]\vec{n}_3 = [\vec{n}_1 \quad \vec{n}_2 \quad \vec{n}_3]\vec{n}_4$

If a plane contains line of intersection of planes $\vec{r} \cdot \vec{n}_1 = q_1, \vec{r} \cdot \vec{n}_2 = q_2$ and is parallel to line of intersection of planes $\vec{r} \cdot \vec{n}_3 = q_3, \vec{r} \cdot \vec{n}_4 = q_4$ then

D. $[\vec{n}_1 \quad \vec{n}_2 \quad \vec{n}_4]\vec{n}_3 = [\vec{n}_1 \quad \vec{n}_2 \quad \vec{n}_3]\vec{n}_4$.

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which of the followings has the value equal to the integral $\int_{1/e}^{\tan x} \frac{t}{1+t^2}\,dt + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$?

A. 1
B. 2
C. $\lim_{x\to 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$
D. $\lim_{n\to\infty} \{\tan\frac{\pi}{2n} \tan\frac{2\pi}{2n} \tan\frac{3\pi}{2n} \ldots\ldots \tan\frac{n\pi}{2n}\}^{1/n}$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let
$I_1 = \int_0^{\frac{\pi}{200}} \sin(2\pi\cos^2 100x)dx, I_2 = \int_0^{\frac{\pi}{200}} \sin(\pi\cos^2 100x)dx, I_3 = \int_0^{\frac{\pi}{200}} \sin(\pi\cos^2 50x)dx$, then

A. $I_1 = 0$
B. $I_2 = 0$
C. $I_1 = I_2$
D. $I_2 = I_3$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

$A(3,4) B(0,0)$ and $C(3,0)$ are vertices of $\triangle ABC$. If ' P ' is a point inside $\triangle ABC$ such that $d(P, BC) \le \min.\{d(P, AB), d(P, AC)\}$, where $d(P, BC)$ represents distance of P from BC, then identify correct statement -

A. if $d(P, BC)$ is maximum, then P is incentre of the triangle.
B. if $d(P, BC)$ is maximum, then P is centroid of the triangle.
C. $\max \cdot d(P, BC) = 1$
D. $\max \cdot d(P, BC) = \frac{4}{3}$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A function $f : R \to R^+$ satisfies
$f(x+y) = f(x) \cdot f(y), \forall x, y \in R, f(0) = 1, f'(0) = 2$, then:

A. $\int_0^{\ln 3} [f(x)e^{-x}]dx = \ln 4.5$ (where [.] denotes greatest integer function)
B. $\lim_{x\to 0} [f(x)]$ does not exist (where [.] denotes greatest integer function)

C. $f^{-1}(x) = \ln \sqrt{x}, \forall x > 0$

D. $f(x) < e^{x^2-4x}$ has infinite solution in $(0, 6)$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If the probability of choosing an integer ' k ' out of $2n$ integers $1, 2, 3, \ldots, 2n$ is inversely proportional to $k^4 (1 \leq k \leq 2n)$. If α is the probability that chosen number is odd and β is the probability that chosen number is even, then

A. $\alpha > \frac{1}{2}$

B. $\alpha > \frac{2}{3}$

C. $\beta < \frac{1}{2}$

D. $\beta < \frac{2}{3}$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let tangent at point P on ellipse cuts directrix corresponding to focus S_1 and S_2 of ellipse at D_1 and D_2 respectively. This tangents cuts auxiliary circle at F_1 and F_2 (where $F_1D_1 < F_2D_1$). Lines $D_1 S_1$ and $D_2 S_2$ meet at Q. If $F_1F_2 =$ length of semi major axis then

A. $D_1Q = D_2Q$

B. $D_1Q \neq D_2Q$

C. $\angle PD_1 S_1 = 30^\circ$

D. $\angle PD_2 S_2 = 60^\circ$

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

If $\begin{bmatrix} 1 & 0 & 0 \\ 6 & 2 & 0 \\ 5 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ x^2 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ 2ax + bx^2 \\ 5x + cx^2 + 3 \end{bmatrix} \forall x \in R$ and $f(x)$ is a differentiable function satisfying $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R, (xy \neq 1)$ and $\lim_{x \to 0} \frac{f(x)}{x} = 2$, then find the value of $\left[\int_0^1 \frac{ax^2+bx+c}{f(1)} dx\right]$, where [.] denotes greatest integer function.

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Let A be $m \times m$ matrix with all elements equal to 1 such that $A^n = 16^{17} A, m, n \in N$, find sum of possible values of n.

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

If n_1 is the number of points on the line $3x + 4y = 5$ which is at distance of $(1 + \sin^2\theta)$ units from $(2, 3)$ and n_2 denotes the number of points on the line $3x + 4y = 5$ which is at distance of $\sec^2\theta + 2\text{cosec}^2\theta$ units from $(1, 3)$, then find the sum of squares of roots of the equations $n_2 x^2 - 6x + n_1 = 0$

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

On Argand plane, let A be the total number of integral points inside the region $|z| < 4$ and B the total number of integral points satisfying $|z - 3i| < 4$ and $\frac{\pi}{6} < (\arg(z - (3i)) < \frac{5\pi}{6}$ then the value of $\frac{A}{B}$ is :

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Let $I = \int_{-3}^{-2} \left(\frac{x^2 - x}{x^3 - 3x + 1}\right)^2 \left(1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right) dx$ then the greatest integer less than the value of I, is

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Two numbers are selected independently at random in the interval $[0, 1]$. If the smaller one is less than $1/3$, then find the probability that the larger one is greater than $3/4$.